An extensional perspective on higher categorical models of linear logic

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∞-categories

Sets $X, Y, \ldots \in \mathrm{Set}$	$\rightsquigarrow$	∞-groupoids $X, Y, \ldots \in \mathcal{S}$ (a.k.a. "spaces" or "homotopy types")
Categories $\mathcal{C}, \mathcal{D} \in \mathrm{Cat}$	$\rightsquigarrow$	∞-categories $\mathcal{C}, \mathcal{D} \in \infty\mathrm{Cat}$
Hom-set $\mathrm{Hom}_{\mathcal{C}}(x, y) \in \mathrm{Set}$	$\rightsquigarrow$	Hom-∞-groupoid $\mathrm{Hom}_{\mathcal{C}}(x, y) \in \mathcal{S}$
Associative and unital composition	$\rightsquigarrow$	Associativity and unitality up to higher coherent isomorphisms
Cat is a $(2, 1)$-category	$\rightsquigarrow$	∞Cat is an ∞-category
Terminal object : $\mathrm{Hom}_{\mathcal{C}}(x, *)$ is a singleton	$\rightsquigarrow$	$\mathrm{Hom}_{\mathcal{C}}(x, *)$ is contractible

- adjunctions
- (co)limits
- kan extensions
- (symmetric) monoidal structures, (co)monoids

We often drop the prefix "∞" for brevity.

∞-categorical semantics of linear logic

Definition ([Ben95])

A *linear-non-linear adjunction* is an adjunction

$$\begin{tikzcd} (\mathcal{M}, \times) \arrow[r, shift left, "L"] \arrow[r, phantom, "\perp"] & (\mathcal{L}, \otimes) \arrow[l, shift left, "M"] \end{tikzcd}$$

between a cartesian category $\mathcal{M}$ and a symmetric monoidal closed category $\mathcal{L}$, where the left adjoint $L : \mathcal{M} \to \mathcal{L}$ is strongly monoidal $L(X \times Y) \simeq LX \otimes LY$.

The functor $LM : \mathcal{L} \to \mathcal{L}$ models the exponential of linear logic.

Example (Lafont exponential)

If $(\mathcal{L}, \otimes, \multimap)$ admits cofree cocommutative comonoids, there is a linear-non-linear adjunction

$$\begin{tikzcd} (\mathrm{Comon}(\mathcal{L}), \times) \arrow[r, shift left] \arrow[r, phantom, "\perp"] & (\mathcal{L}, \otimes). \arrow[l, shift left] \end{tikzcd}$$

Intensional and extensional perspectives

Intensional	Extensional
$\{x^2 + 4x + 1 \mid x \in \mathbb{Z}\}$	$\{x \in \mathbb{N} \mid x + 3 \text{ is a perfect square}\}$
Matrix $(a_{ij}) \in M_{m,n}(k)$	Linear map $k^m \to k^n$

Categorically:

category	k-Mat	k-Vect
objects	natural numbers m, n	finite-dimensional k-vector spaces E, F
morphisms	matrices $M \in M_{m,n}(k)$	linear maps $E \to F$

The functor $m \mapsto k^m : k\text{-Mat} \to k\text{-Vect}$ is an equivalence of categories.

Summary

dimension	0-categories ($=$ posets)	∞-categories
intensional category	Porel	∞Prof
objects	posets	∞-categories
morphisms	posetal relations $E \times F^{\mathrm{op}} \to \mathrm{Bool}$	profunctors $\mathcal{C} \times \mathcal{D}^{\mathrm{op}} \to \mathcal{S}$
extensional category	SupLat	$\infty\mathrm{Cat}_{\mathrm{cc}}$
objects	(co)complete lattices	cocomplete ∞-categories
morphisms	arbitrary join preserving maps	cocontinuous functors
free exponential	multisets Mul	free symmetric monoidal ∞-category Sym
non-linear maps	“boolean polynomials”	generalized ∞-species
Scott exponential	finite join completion	finite colimit/coproduct cocompletion
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The category of posets and posetal relations

Definition

The category Porel of posets and posetal relations has

- objects: posets
- morphisms: relations $R \subseteq X \times Y$ such that $\forall x, x', y, y'$,

$$x' \geq x \; R \; y \geq y' \implies x' \; R \; y'$$

  or equivalently, $R : X \times Y^{\mathrm{op}} \to \mathrm{Bool}$
- cartesian product: disjoint union of underlying posets $X \sqcup Y$
- tensor product: cartesian product of underlying posets $X \otimes Y$

The category of suplattices

Definition

The category SupLat of suplattices has

- objects: complete lattices $(E, \leq)$
- morphisms: monotonous maps $f : E \to F$ that **preserve all joins**
- cartesian products: cartesian product of underlying posets
- a tensor product such that

$$\mathrm{Hom}_{\mathrm{SupLat}}(E \otimes F, G) \simeq \mathrm{Bilin}(E \times F, G)$$

where $f : E \times F \to F$ is in $\mathrm{Bilin}(E \times F, G)$ if and only if it preserves joins separately in both variables.

The extensional perspective on posetal relations

Every poset X defines a suplattice $P(X)$ of **downards-closed subsets of** X called the **powerset** of X.
Every posetal relation $R \subset X \times Y$ induces a suplattice morphism

$$\begin{aligned} P(X) &\to P(Y) \\ U \subseteq X &\mapsto \{y \in Y \mid \exists x \in U, x \, R \, y\} \end{aligned}$$

Proposition

The induced functor $P(-) : \mathrm{Porel} \to \mathrm{SupLat}$ is fully faithful. In other words, the full subcategory of SupLat on powerset lattices is equivalent to Porel.

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The multiset exponential on relations and suplattices

The multiset functor $\mathrm{Mul} : \mathrm{Poset} \to \mathrm{Poset}$ lifts to an exponential comonad $\mathrm{Mul} : \mathrm{Porel} \to \mathrm{Porel}$.

Proposition

Given a poset X, $\mathrm{Mul}(X)$ is the cofree cocommutative comonoid on X in Porel.

Proposition

SupLat admits cofree cocommutative comonoids, and $P : \mathrm{Porel} \to \mathrm{SupLat}$ preserves them.

Hence SupLat extends Porel as a model of linear logic.

The Scott exponential on SupLat and Porel

Definition

Scott the category of posets with joins of **directed families** and monotonous maps that preserve directed joins.

There is a chain of symmetric monoidal left adjoints

$$\begin{tikzcd} \mathrm{Set} \arrow[r, shift left] \arrow[r, phantom, "\perp"] & \mathrm{Poset} \arrow[l, shift left] \arrow[r, shift left] \arrow[r, phantom, "\perp"] & \mathrm{Scott} \arrow[l, shift left] \arrow[r, shift left] \arrow[r, phantom, "\perp"] & \mathrm{SupLat} \arrow[l, shift left] \end{tikzcd}$$

where the monoidal structures on $\mathrm{Set}, \mathrm{Poset}$ and Scott are cartesian.

$\rightsquigarrow$ 3 exponential comonads on SupLat that restrict to Porel as follows: $!_{\mathrm{Set}}, !_{\mathrm{Poset}}$ and $!_{\mathrm{Scott}}$.

- $!_{\mathrm{Set}}(E) = P(|E|)$ the powerset on the underlying set of E.
- $!_{\mathrm{Poset}}(E) = P(E)$ the completion of E under arbitrary joins.
- $!_{\mathrm{Scott}}(E) =$ the completion of E under finite joins.

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Categorifying relations

Going from 0-categories (posets) to ∞-categories :

$$\text{posetal relations} \rightsquigarrow \infty\text{-profunctors}$$

Definition

Let $\mathcal{C}, \mathcal{D}$ be ∞-categories.
A profunctor $F : \mathcal{C} \nrightarrow \mathcal{D}$ is a functor $F : \mathcal{C} \times \mathcal{D}^{\mathrm{op}} \to \mathcal{S}$.

Not clear how to define an ∞-category of ∞-categories and profunctors: composition is usually defined using coends, and associativity is shown by hand using coend computations.
Getting higher coherences seems impossible using this approach.

The extensional perspective on profunctors

$$\dfrac{\dfrac{\dfrac{\mathcal{C} \times \mathcal{D}^{\mathsf{op}} \to \mathcal{S}}{\mathcal{C} \to \mathsf{Fun}(\mathcal{D}^{\mathsf{op}}, \mathcal{S})}}{\mathcal{C} \to \mathcal{P}(\mathcal{D})}}{\mathcal{P}(\mathcal{C}) \to_{\mathsf{cc}} \mathcal{P}(\mathcal{D})}\ \text{(kan extension along Yoneda)}$$

Hence profunctors correspond to cocontinuous functors between presheaf ∞-categories.

Definition

Write $\infty\mathsf{Cat}_{\mathsf{cc}}$ for the ∞-category of ∞-categories with small colimits and functors that preserve them (called *cocontinuous functors*).
Write $\infty\mathsf{Prof}$ for its full subcategory on the presheaf ∞-categories.

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Monoidal structures on ∞-categories of ∞-categories

Definition

Given a class $\mathbb{K}$ of diagrams, write $\infty\mathrm{Cat}_{\mathbb{K}}$ for the subcategory of $\infty\mathrm{Cat}$ on ∞-categories that admit colimits indexed by diagrams in $\mathbb{K}$ and functors that preserve those colimits.

- symmetric monoidal closed structure on $\infty\mathrm{Cat}_{\mathbb{K}}$
- monoidal left adjoints to forgetting of colimits when $\mathbb{K} \subset \mathbb{K}'$

$$(\infty\mathrm{Cat}_{\mathbb{K}}, \otimes) \underset{\longleftarrow}{\overset{\longrightarrow}{\perp}} (\infty\mathrm{Cat}_{\mathbb{K}'}, \otimes)$$

Cocompletion-based exponentials

Example

- $\infty\mathsf{Cat}_\emptyset = \infty\mathsf{Cat}$
- $\infty\mathsf{Cat}_{\mathsf{cc}}$ for $\mathbb{K}$ = all (small) diagrams ("cc" for cocontinuous)
- $\infty\mathsf{Cat}_{\mathsf{filtr}}$ for *filtered* diagrams
- $\infty\mathsf{Cat}_{\mathsf{sift}}$ for *sifted* diagrams

"Filtered" and "sifted" diagrams are different categorical generalizations of *directed posets*.

$$\begin{tikzcd}
\mathcal{S} \arrow[r, bend left] \arrow[r, phantom, "\perp"] & \infty\mathsf{Cat} \arrow[l, bend left] \arrow[r, bend left] \arrow[r, phantom, "\perp"] & \infty\mathsf{Cat}_{\mathsf{filtr}} \arrow[l, bend left] \arrow[r, bend left] \arrow[r, phantom, "\perp"] & \infty\mathsf{Cat}_{\mathsf{sift}} \arrow[l, bend left] \arrow[r, bend left] \arrow[r, phantom, "\perp"] & \infty\mathsf{Cat}_{\mathsf{cc}} \arrow[l, bend left]
\end{tikzcd}$$

$$\begin{tikzcd}
\mathsf{Set} \arrow[r, bend left] \arrow[r, phantom, "\perp"] & \mathsf{Poset} \arrow[l, bend left] \arrow[r, bend left] \arrow[r, phantom, "\perp"] & \mathsf{Scott} \arrow[l, bend left] \arrow[rr, bend left] \arrow[rr, phantom, "\perp"] & & \mathsf{SupLat} \arrow[ll, bend left]
\end{tikzcd}$$

Cocompletion-based exponentials (2)

$$\begin{tikzcd} \infty\mathsf{Cat}_{\mathsf{filtr}} \arrow[r, bend left] \arrow[r, phantom, "\perp"] & \infty\mathsf{Cat}_{\mathsf{cc}} \arrow[l, bend left] \end{tikzcd} \qquad !_{\mathsf{filtr}}X = \text{ cocompletion under finite colimits}$$

$$\begin{tikzcd} \infty\mathsf{Cat}_{\mathsf{sift}} \arrow[r, bend left] \arrow[r, phantom, "\perp"] & \infty\mathsf{Cat}_{\mathsf{cc}} \arrow[l, bend left] \end{tikzcd} \qquad !_{\mathsf{sift}}X = \text{ cocompletion under finite coproducts}$$

$$\begin{tikzcd} \mathsf{Scott} \arrow[r, bend left] \arrow[r, phantom, "\perp"] & \mathsf{SupLat} \arrow[l, bend left] \end{tikzcd} \qquad !_{\mathsf{Scott}}X = \text{ cocompletion under finite joins}$$

What about the free exponential ?

Free exponential on Prof

Theorem ([Lur17])

Let $(\mathcal{C}, \otimes)$ with countable colimits. If $X \otimes - : \mathcal{C} \to \mathcal{C}$ preserves small colimits, $\mathcal{C}$ admits free commutative monoids, given by

$$A \mapsto \coprod_{n \in \mathbb{N}} A^{\otimes n} // \mathfrak{S}_n$$

This applies to $\infty\mathrm{Cat}_{cc}$!

Theorem

$$\mathcal{C} \mapsto \coprod_{n \in \mathbb{N}} \mathcal{C}^{\otimes n} // \mathfrak{S}_n$$

is the free commutative monoid on $\mathcal{C}$ in $\infty\mathrm{Cat}_{cc}$.

Free exponential on Prof (2)

$\mathcal{P} : \infty\mathrm{Prof} \to \infty\mathrm{Cat}_{\mathrm{cc}}$ preserves free commutative monoids.

$$\coprod_{n\in\mathbb{N}} \mathcal{P}(\mathcal{C})^{\otimes n}//\mathfrak{S}_n \quad \simeq \quad \mathcal{P}(\mathrm{Sym}(\mathcal{C}))$$

$\rightsquigarrow \infty\mathrm{Prof}$ admits free commutative monoids.

$\infty\mathrm{Prof}$ is self-dual by $\mathcal{C} \mapsto \mathcal{C}^{\mathrm{op}}$, so it has cofree cocommutative comonoids

$$\mathcal{C} \mapsto (\mathrm{Sym}(\mathcal{C}^{\mathrm{op}}))^{\mathrm{op}} \simeq \mathrm{Sym}(\mathcal{C})$$

Non-linear maps are generalized species

Non-linear maps:

$$\mathrm{Sym}(\mathcal{C}) \times \mathcal{D}^{\mathrm{op}} \to \mathcal{S}$$

"generalized ∞-species of structure" [Fio+08]

In 1-categories, $F : \mathrm{Sym}(\mathcal{C}) \times \mathcal{D}^{\mathrm{op}} \to \mathrm{Set}$ corresponds to **analytic functor** [Fio13]

$$\mathcal{P}(\mathcal{C}) \to \mathcal{P}(\mathcal{D})$$

Bonus: recovering spans from profunctors

Given groupoids X, Y, a 1-profunctor $X \nrightarrow Y$ is a functor $X \times Y \to \mathrm{Set}$, which corresponds to a discrete fibration $Z \to X \times Y$.

X, Y ∞-groupoids. ∞-profunctor $X \nrightarrow Y$ = functor $X \times Y \to \mathcal{S}$, which corresponds to an arbitrary functor $Z \to X \times Y$, or in other words a span

$$X \longleftarrow Z \longrightarrow Y$$

∞-profunctors between ∞-groupoids correspond exactly to spans of ∞-groupoids.
$\rightsquigarrow$ ∞-categorical span model of linear logic (earlier span models include [Mel19; CF23])

Conclusion

- The extensionnal approach to relations tends itself very well to formal generalizations in higher-categorical settings.
- Leveraging the existing theory of cocompletions in ∞-categories, we found ∞-categorical analogues to the Scott exponential of linear logic.
- Using the theory of free monoids in ∞-categories, we also built the free exponential on profunctors.
- This way we defined an ∞-category of generalized species.
- While defining the usual bicategory of species involves heavy coend computations, in ∞-categories we had to work more abstractly using the extensional perspective: computational behavior can only be recovered afterward.

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